Wave Compression in Plasma

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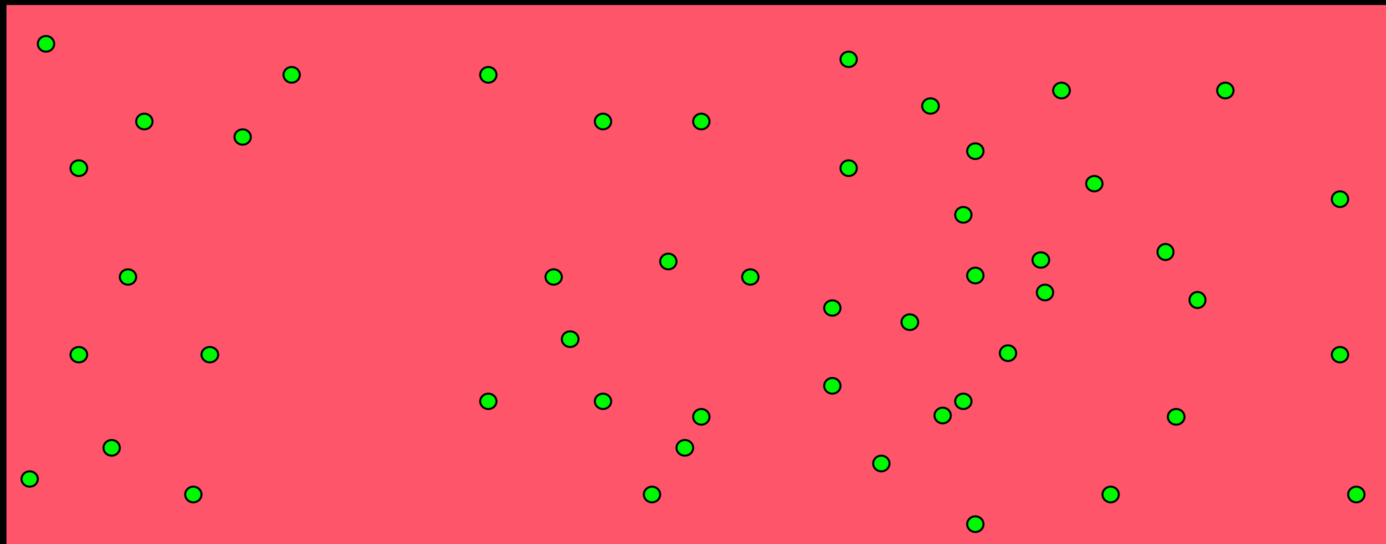
Colloquium at Fermilab, July 10, 2014

Simple wave oscillations in plasmas can produce enormous effects. This talk first explains what is a plasma wave. I then focus on two recently discovered, curious, and potentially useful effects, both mediated by the plasma wave, and both involving wave compression. One effect is resonant Raman backscattering, whereby a long moderately intense laser beam loses its energy to a short counter-propagating beam, producing a much shorter and much more intense pulse. This effect might overcome the material limitations of present technology, enabling the next generation of laser intensities. A second compression effect occurs when the plasma itself is compressed; not only does its temperature increase, but any embedded waves might also increase in energy. For adiabatic changes in time of the density of the plasma medium, the coherent wave energy grows, but, importantly, might then very abruptly lose this energy.

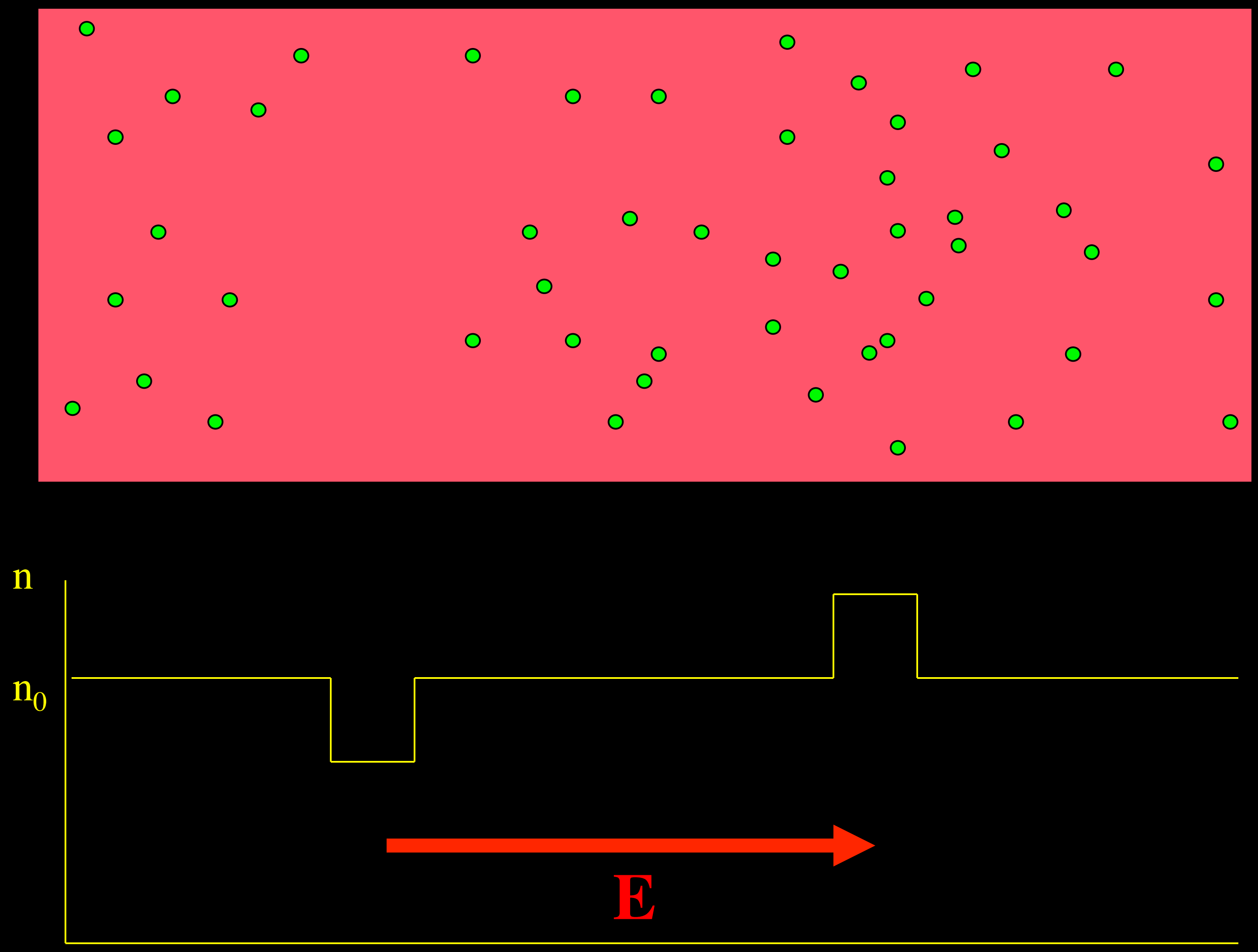
Plasma



In 1927, Irving Langmuir coined the term *plasma* for an ionized gas, because an electrified fluid carrying ions and electrons reminded him of how blood plasma carried red and white corpuscles.

Irving Langmuir (1881-1957); Nobel '32

Set up plasma oscillation (Langmuir wave)



Plasma Oscillations

$$\nabla \bullet \vec{E} = 4\pi e(n_0 - n_e) = -4\pi e\tilde{n}$$ Poisson's equation $$n_e = n_0 + \tilde{n}$$

$$\frac{\partial}{\partial t} n_e + \nabla \cdot n_e \vec{v} = 0$$ Particle conservation $$\vec{E} = \tilde{\vec{E}}$$

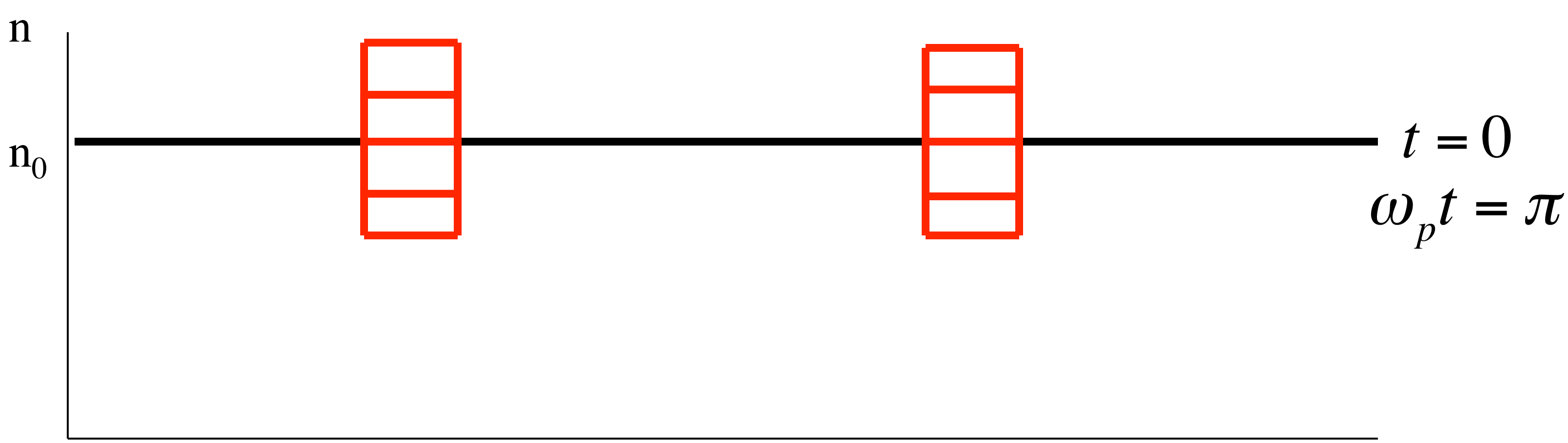
$$\frac{\partial}{\partial t} n_e m \vec{v} + \nabla \cdot n_e m \vec{v}\,\vec{v} = e n_e \vec{E}$$ Momentum conservation $$\vec{v} = \tilde{\vec{v}}$$

$$\Rightarrow \quad \frac{\partial^2}{\partial t^2}\tilde{n} + \omega_p^2 \tilde{n} = 0$$

$$\omega_p^2 = 4\pi e^2 n_0 / m$$

$$\boxed{\tilde{n} = A(\vec{r})\cos\omega_p t + B(\vec{r})\sin\omega_p t}$$

Plasma Oscillation



Make traveling wave:

$$\boxed{\Phi(x,t) = A(x)\cos[\omega_p\,(t - x/c)]}$$

Resonant Surfers

Not-resonant surfers $V \neq V_{ph}$

Things that a plasma wave can do

1. Toroidal current in tokamaks (3 MA)
2. High-gradient accelerators (100 GeV)

Significant Progress

1. Mediate resonant Raman compression of optical lasers
2. Compression of x-rays, short wavelength optical

Promising

Goal: Achieve next generation of light intensities

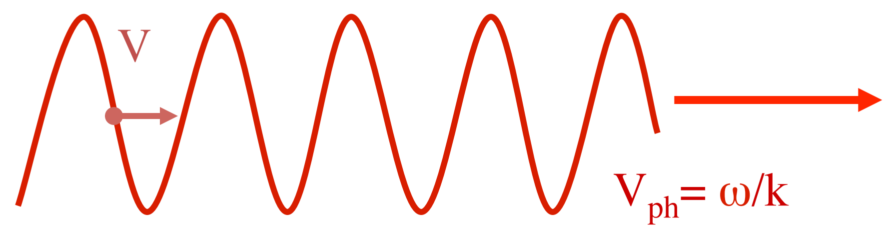
1. Switch-like mechanisms in compression of plasma waves
2. Couple diffusion in space to diffusion in energy – cooling effect

Speculative

Goal: Realize new effects in new facilities for highly compressing plasma

Accelerating Gradient in Plasma

Conventional Accelerator
20 MeV/m at 3 GHz
Limited by breakdown
1 TeV Collider requires 50 km



$$\nabla \bullet \vec{E} = -4\pi e\tilde{n}$$

$$\tilde{n}_{MAX} \approx n_0$$

$$k = \frac{\omega_p}{c}$$

$$eE_{MAX} \approx \sqrt{n_0}\, GeV/cm$$

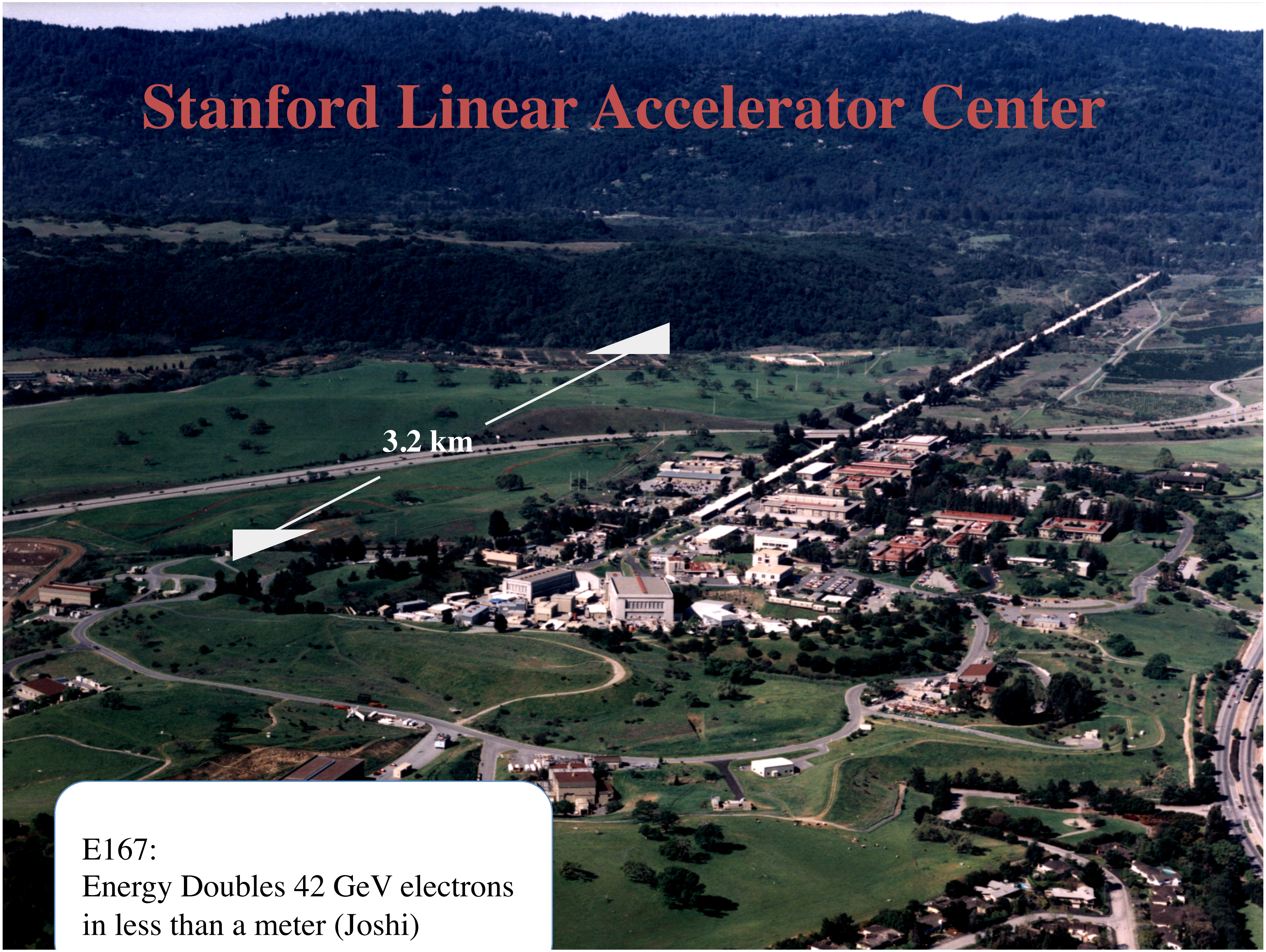
Plasma Accelerator
High field gradients (10^4)

$n_0 = 10^{18}\ \mathrm{cm}^{-3}$

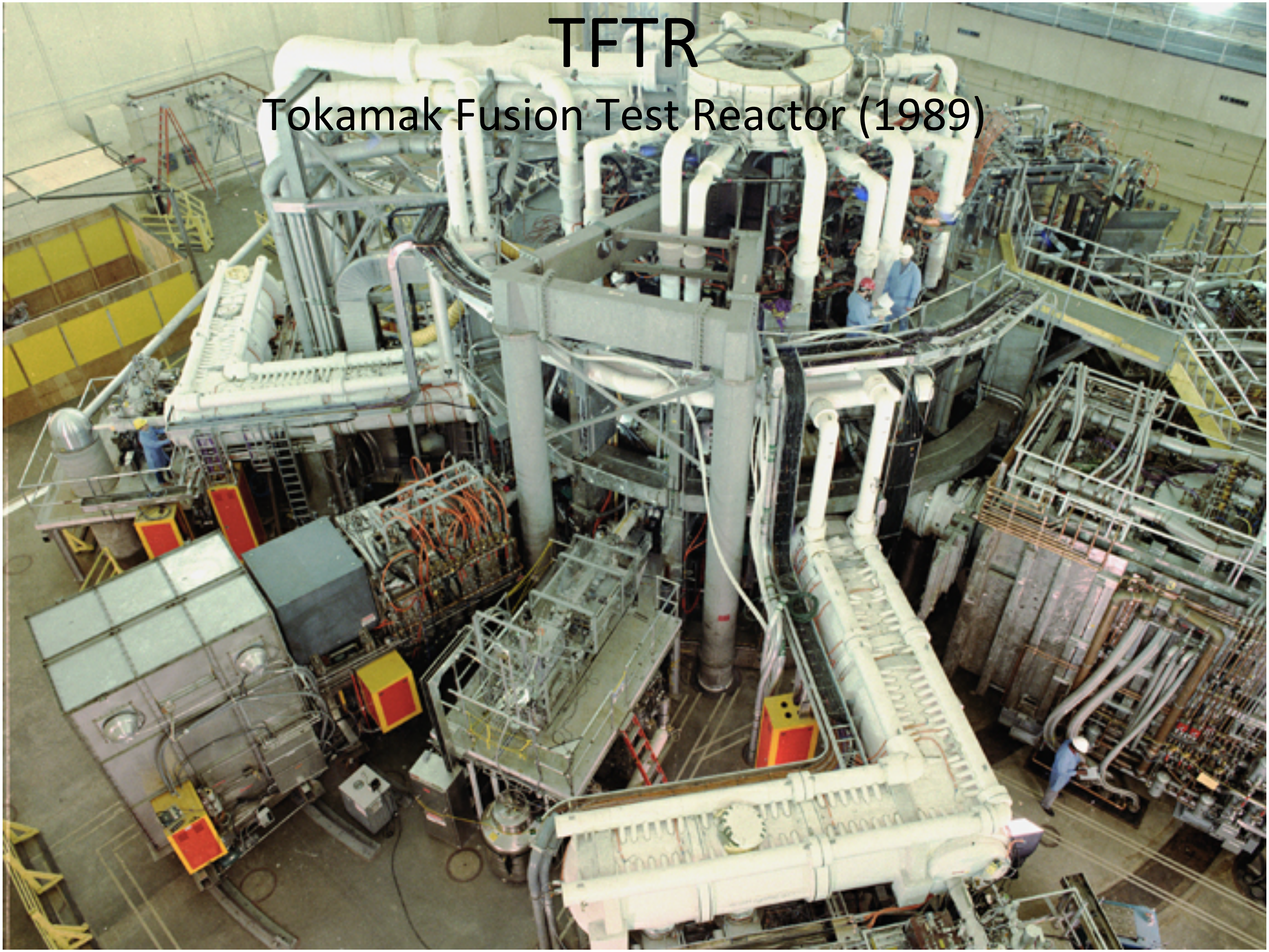
$eE = 100$ GeV/m

Note: For v << c , $\dfrac{v_{osc}}{c} \approx \dfrac{\tilde{n}}{n_0}$

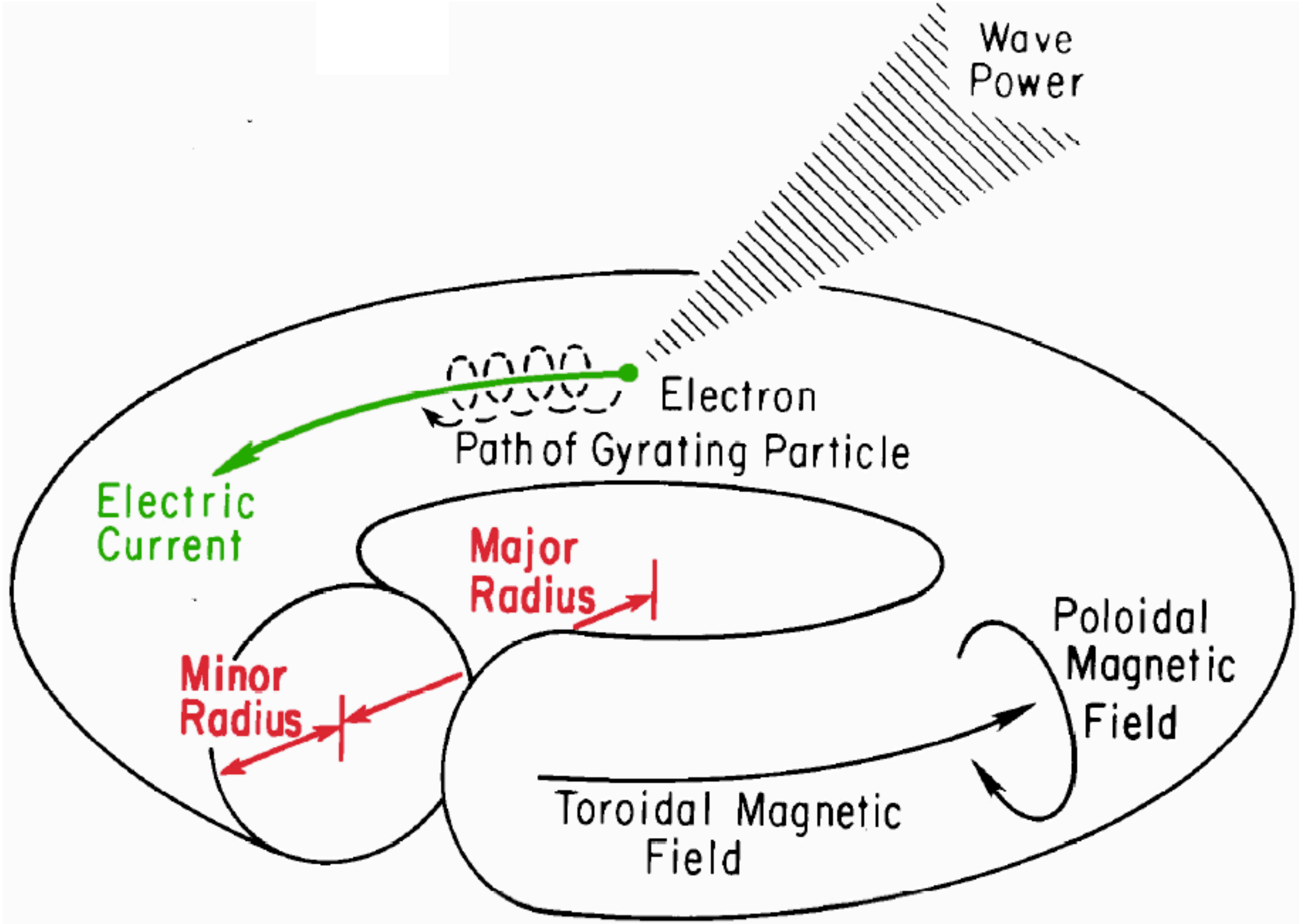
Particles accelerated to relativistic energies, even as plasma motion is not


Stanford Linear Accelerator Center
3.2 km
E167:
Energy Doubles 42 GeV electrons
in less than a meter (Joshi)

TFTR
Tokamak Fusion Test Reactor (1989)



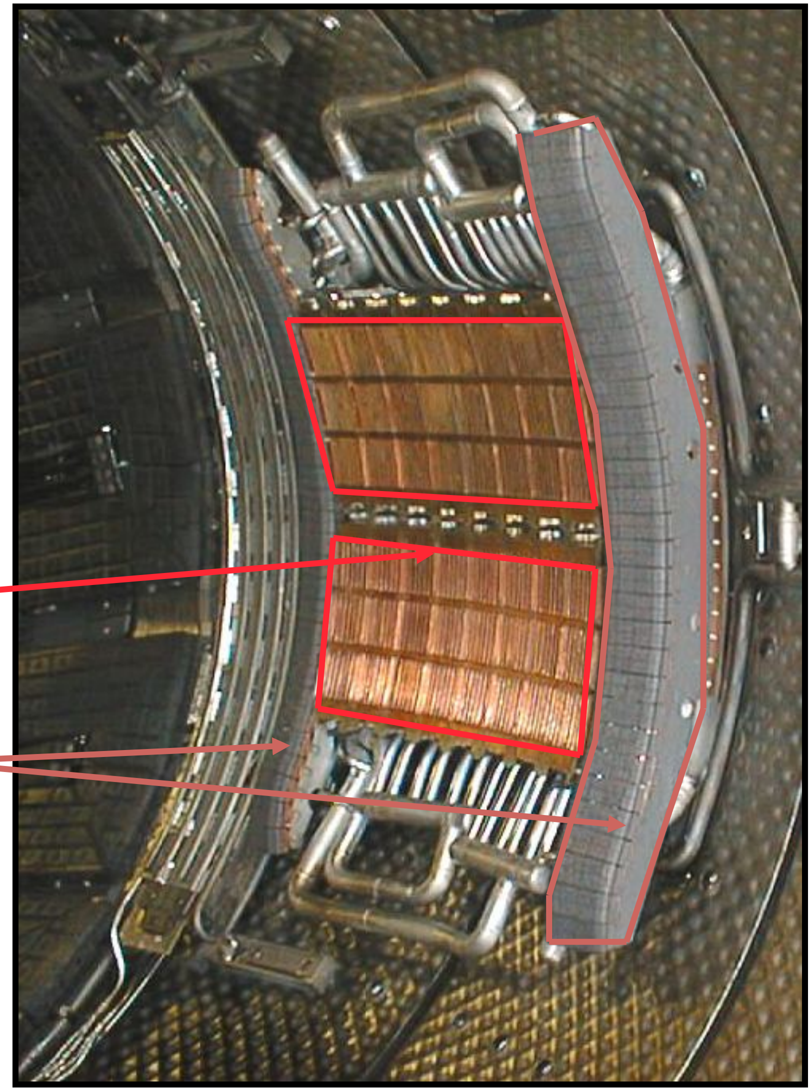
Generating Current with Waves



Tore Supra: LH coupler

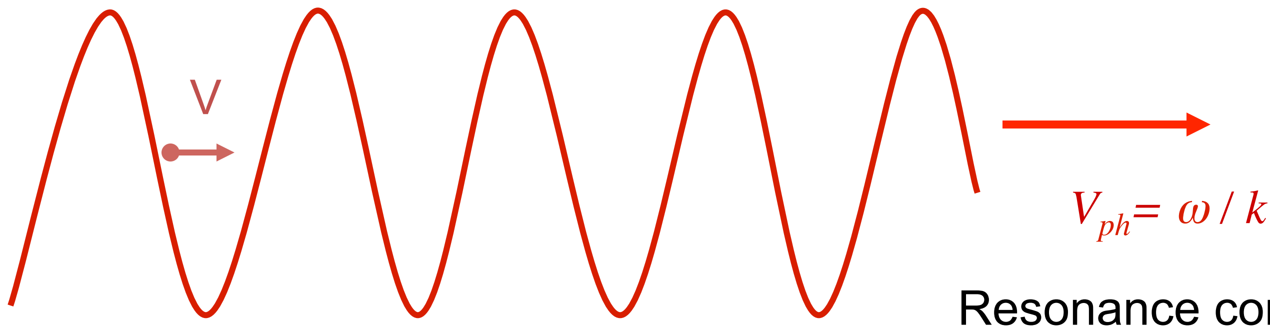
Antenna for long pulse operation:
4 MW, 1000 s, 3.7 GHz
(25 MW/m^2 and $n_{//0}$ = 2)

Actively cooled side limiter
(exhaust capability:10 MW/m^2)



48 active, 9 passive waveguides

Radio Frequency (RF) Current Drive Effect



Resonance condition

$$\omega - \vec{k} \cdot \vec{v} = 0$$

$$\mathrm{v} \longrightarrow \mathrm{v} + \Delta \mathrm{v}$$

$$J = en\Delta \mathrm{v}$$

$$\Delta E = mn\mathrm{v}\Delta \mathrm{v}$$

$$P_{\mathrm{D}} = \nu \Delta E$$

$$\nu(\mathrm{v}) \approx \mathrm{v}^{-3}$$

$$\boxed{J/P_{\mathrm{D}} = e/m\mathrm{v}\nu(\mathrm{v})}$$

$\omega - k_{\parallel}\mathrm{v}_{\parallel} = 0 \Longrightarrow$ LHCD, FWCD, MCIBW

Fisch (1978)

$\omega - k_{\parallel}\mathrm{v}_{\parallel} - n\Omega = 0 \Longrightarrow$ ECCD

Fisch and Boozer (1980)

Examples: RF Current Drive

JT-60 and JT60-U (Japan) -- 3MA LHCD 800 kA ECCD, ITB sawteeth stabilization (2001)

JET (England) -- 3 MA LHCD, ITB with LHCD, Minority Species CD. ITB

Tore Supra (France) -- 1000 s LHCD, ITB; 330 s, 1 GJ, LHCD (2004), ECCD Synergy

C-Mod tokamak (MIT) : LHCD

TRIAM (Japan): several hours LHCD

T-10 (Russia): ECCD , sawteeth

TCV tokamak --- ECCD steady state, sawteeth

ASDEX (Germany): ECCD stabilization of tearing modes

Wendelstein 7-AS Stellarator: ECCD

Frascati FT-U (Italy): LHCD, ECCD stabilization of sawteeth, tearing modes

General Atomics DIIID tokamak; ECCD, ITB, mode suppression

Princeton spherical torus: NSTX (HHFWCD)

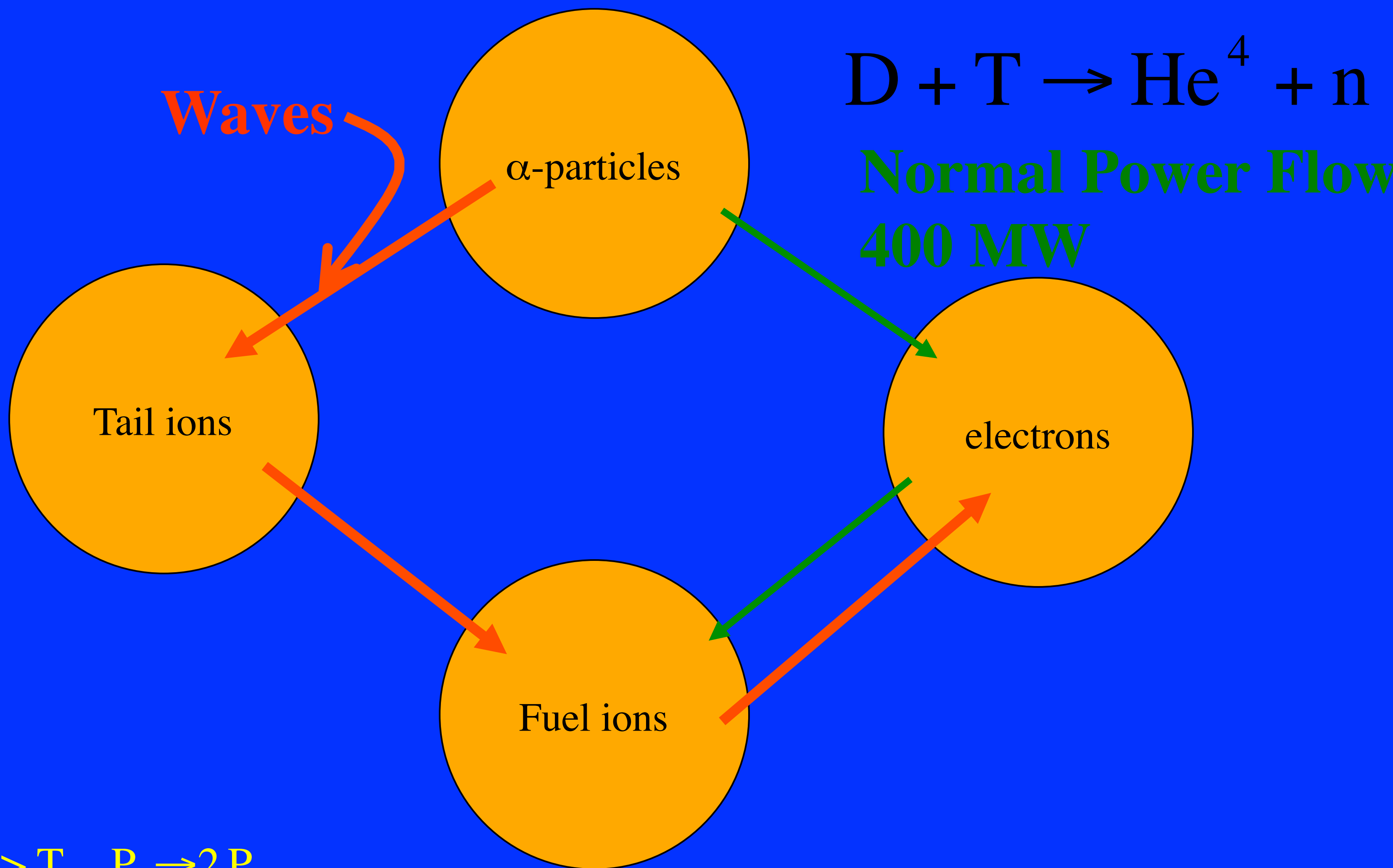
New Steady-State Lower-hybrid driven Superconducting Tokamaks

SST (India)

KSTAR (Korea)

East (China)

The Self Sustaining Fusion Reaction
Power Flow in a Fusion Reactor -- or a *hot spot*



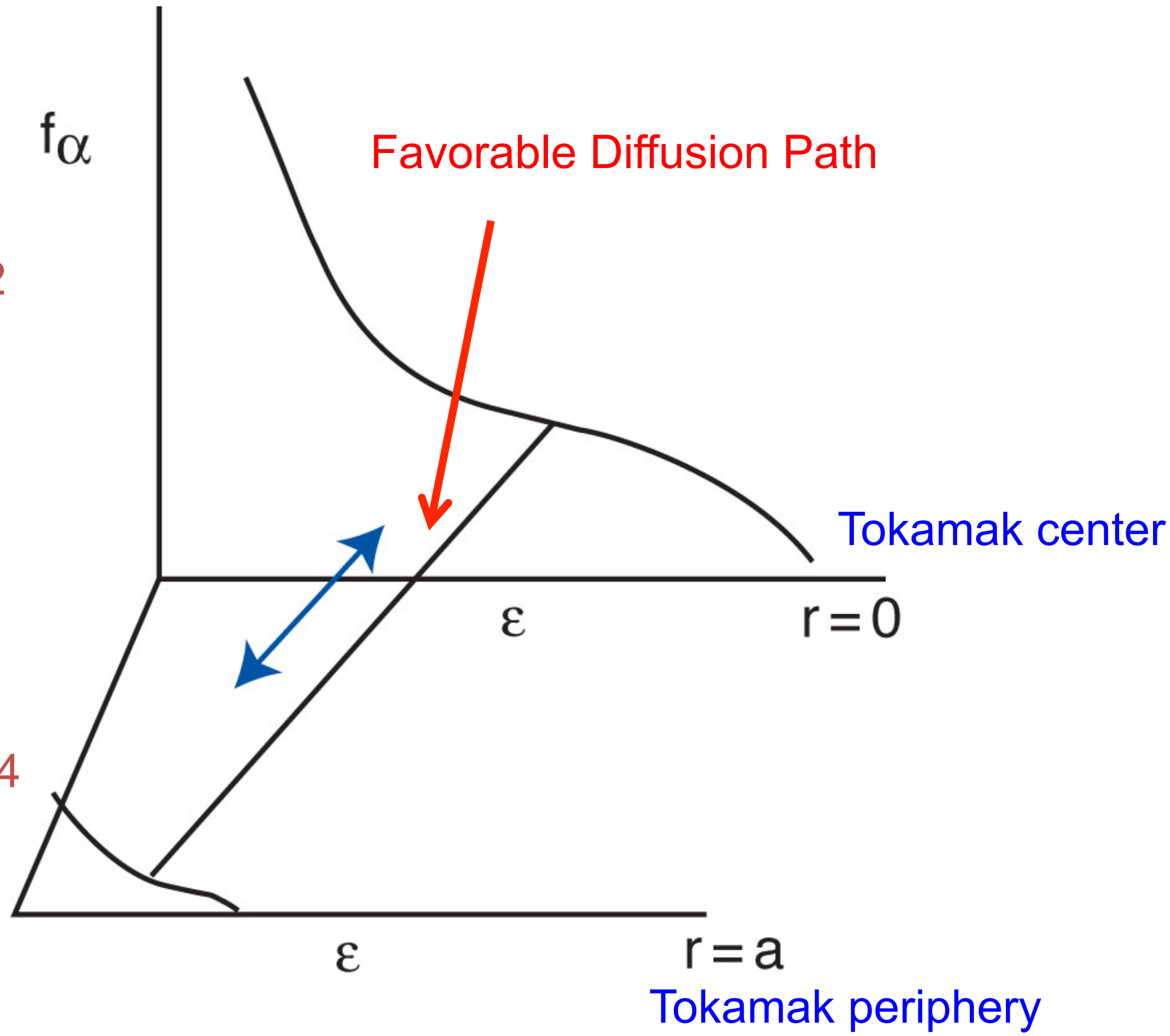
Get $T_i > T_e$, $P_f \rightarrow 2\,P_f$

Fisch and Rax, 1992

Extracting Free Energy

Fisch and Rax, 1992

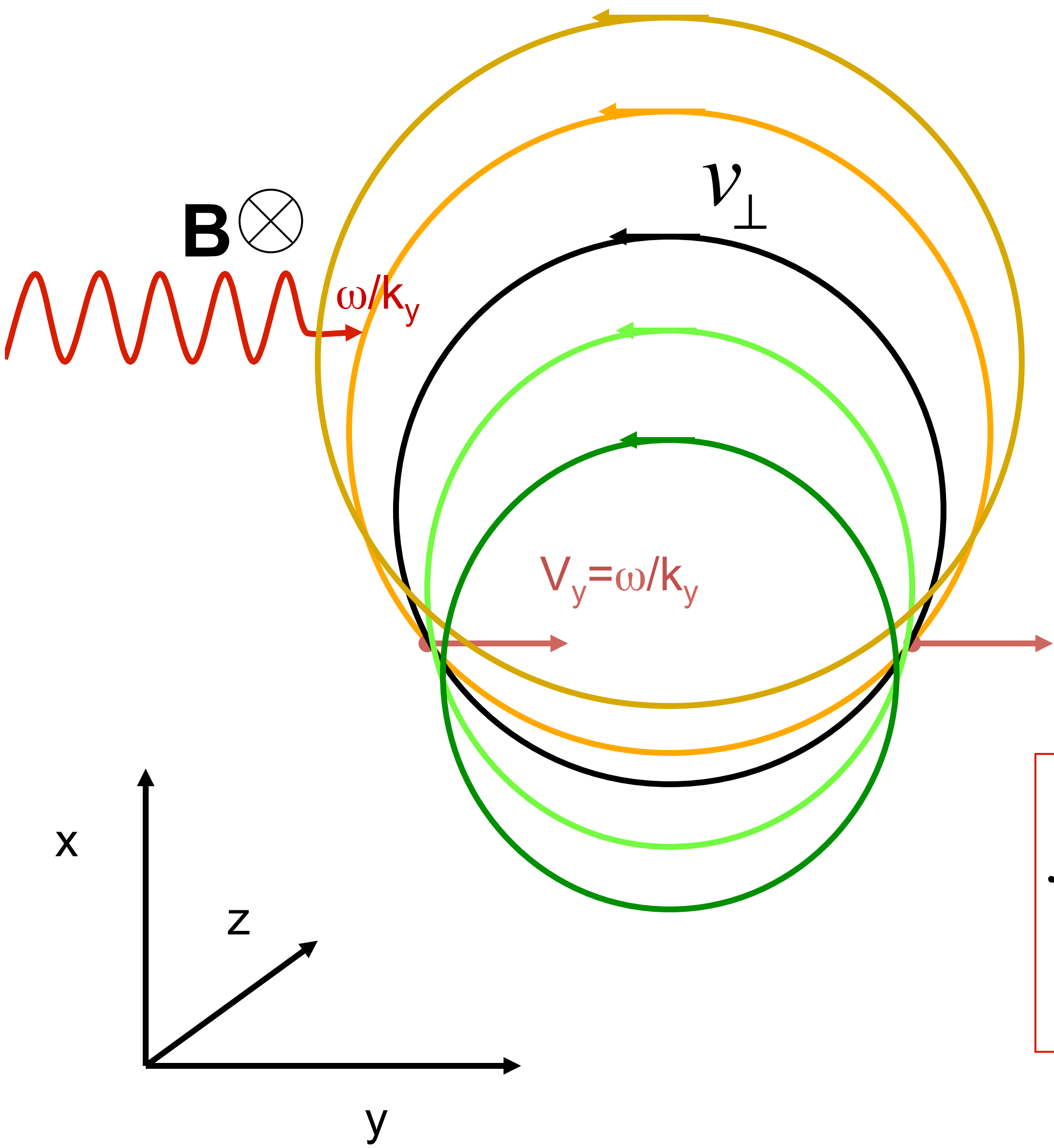
motivated by:
Wong and Ono, 1984



cf. Cook Chapman and Dendy, 2010

Constraining the Diffusion Path

Fisch and Rax, 1992



$$v_y \rightarrow v_y + \Delta v_y$$

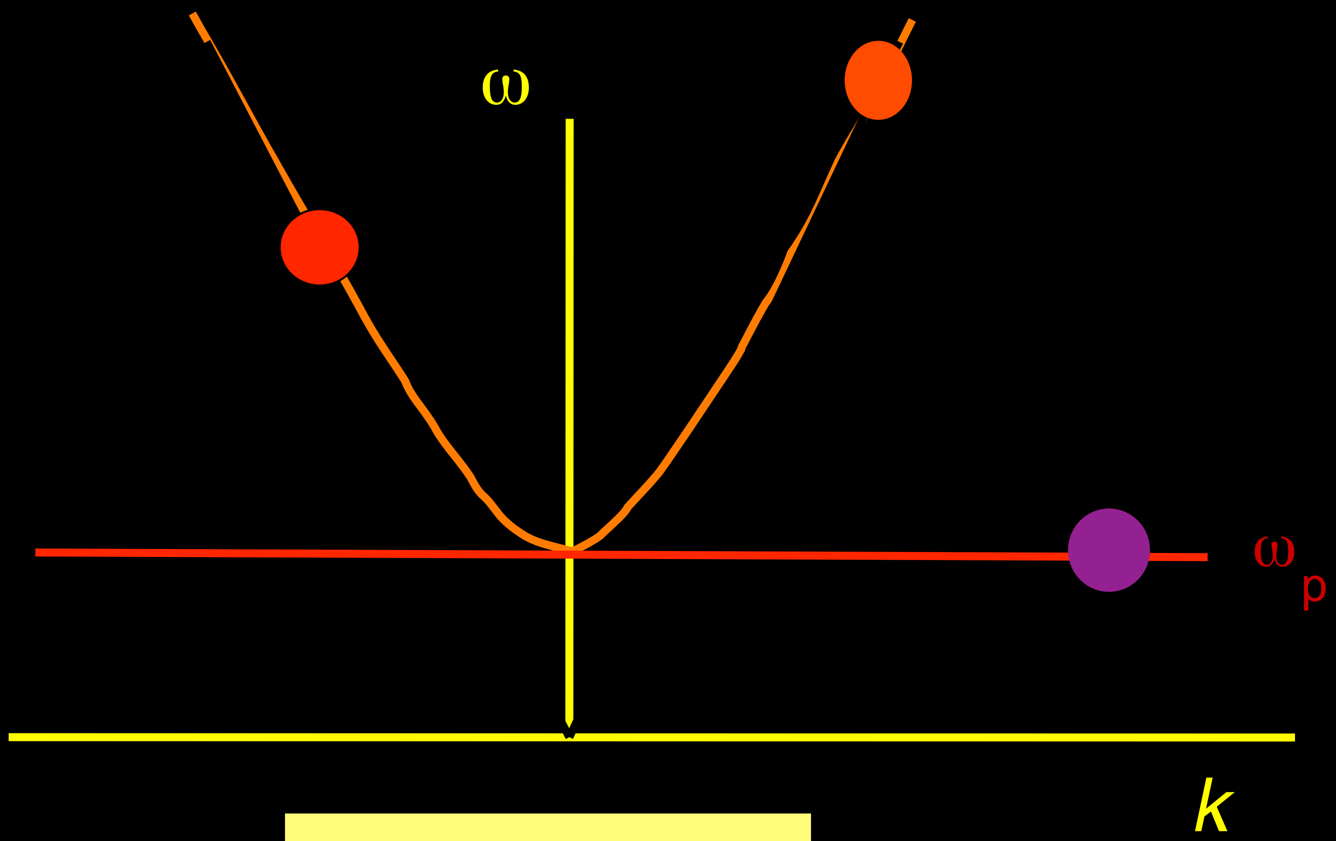
$$x_{gc} \rightarrow x_{gc} + \Delta v_y / \Omega$$

$$\Omega \equiv eB/m$$

$$\Delta E = m v_y \Delta v_y$$

$$\boxed{x_{gc} \rightarrow x_{gc} + \frac{\Delta E}{m\Omega \, \omega / k_y}}$$

Raman Decay in Plasma

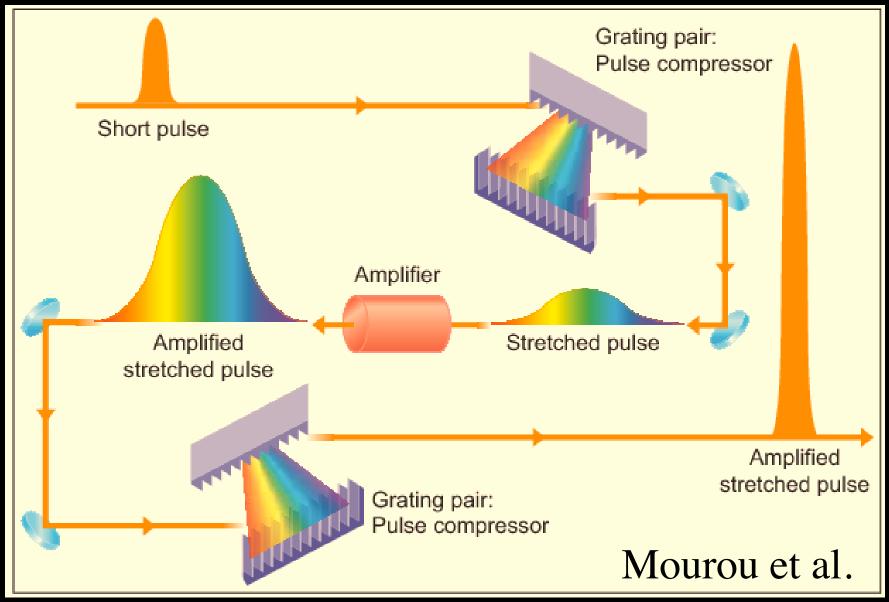


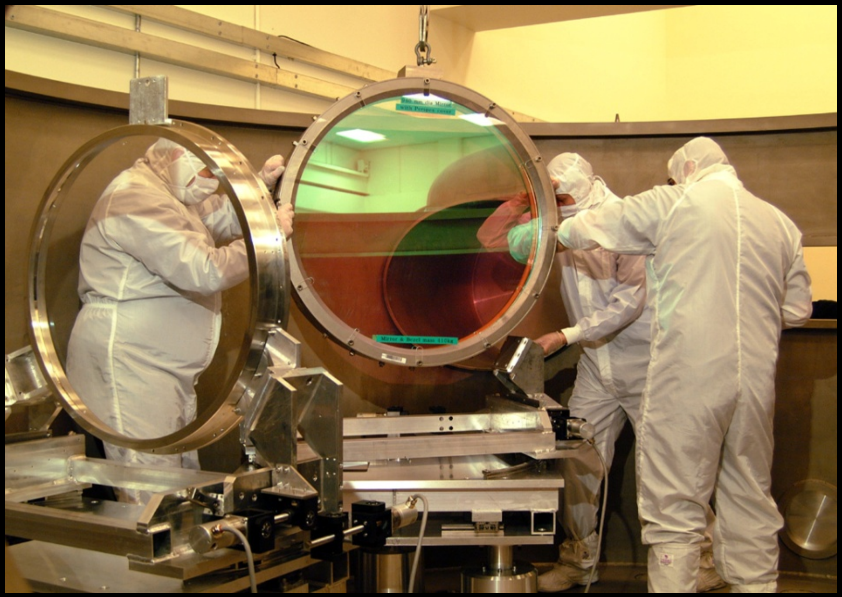
resonance condition

$$\omega_a - \omega_b = \omega_p$$

$$\vec{k}_a - \vec{k}_b = \vec{k}_p$$

Chirped Pulse Amplification: stretch, amplify, then recompress





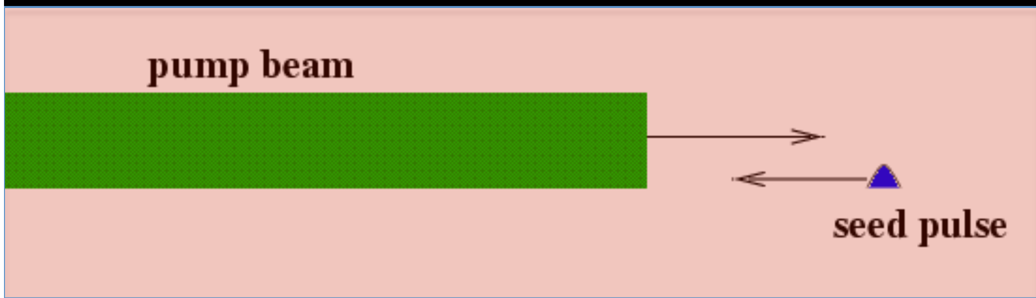
Gratings for Petawatt (10^{15}W) Laser

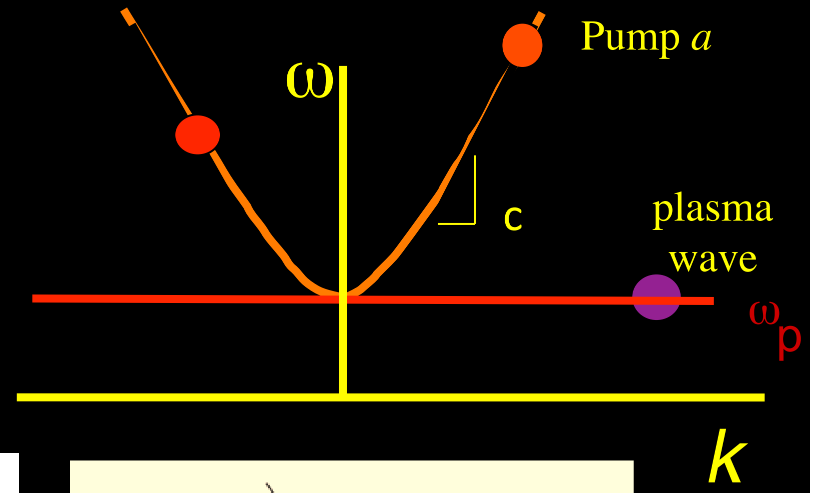
Limitations of CPA

- Thermal damage to expensive gratings
- Requires broad-bandwidth high-fluence amplifiers

GW/cm^2 in amplifier → 10^3 compression → TW/cm^2 < 10 ps → For PW → 10^3 cm^2 gratings

Resonant Raman Amplification and Compression





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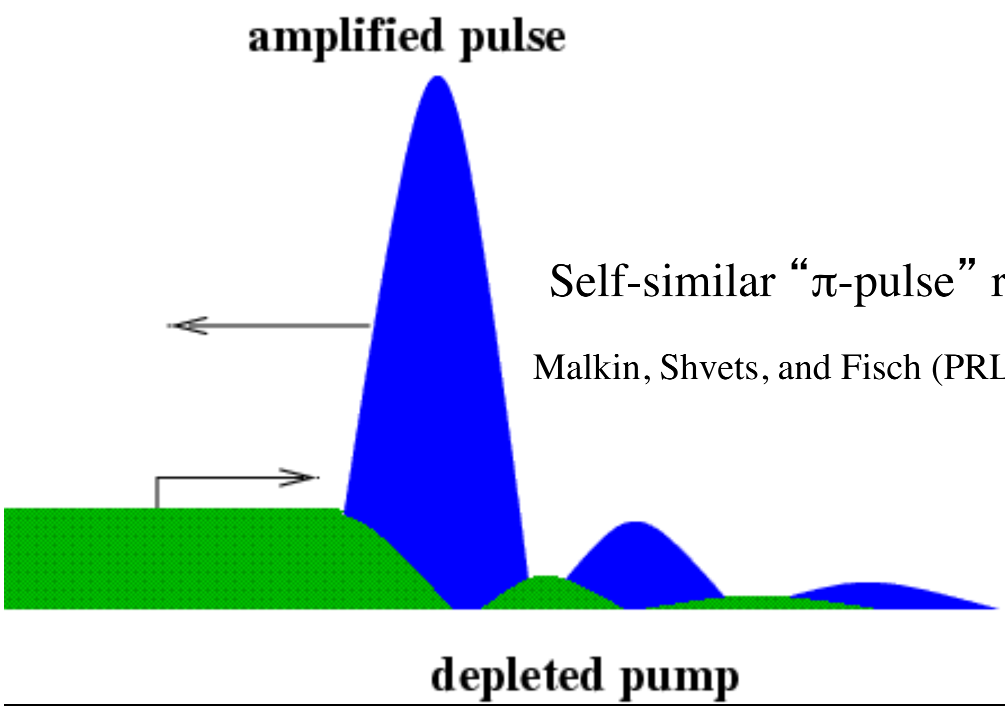
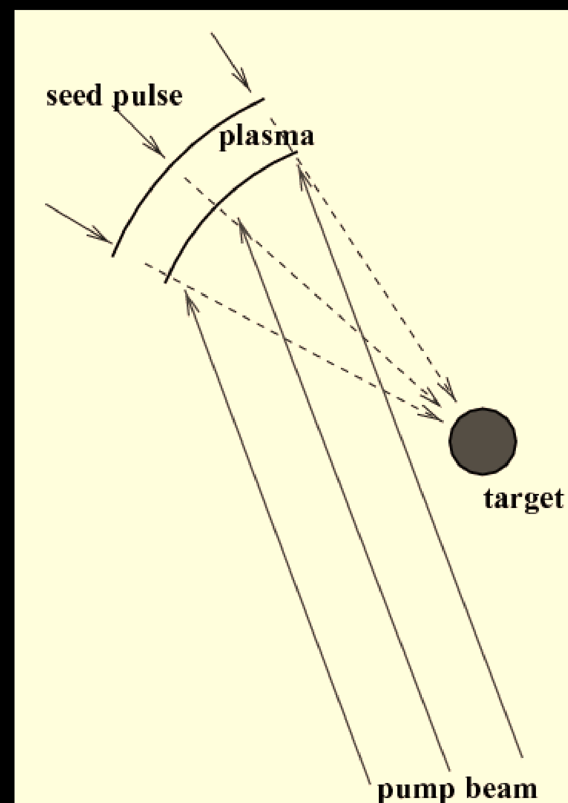


Self-similar "π-pulse" regime

Malkin, Shvets, and Fisch (PRL, 1999)



Fast Compression By RBS

$$a_t + c a_z = -V f b \,,$$

$$f_t = V a b^*$$

$$b_t - c b_z = V a f^*$$

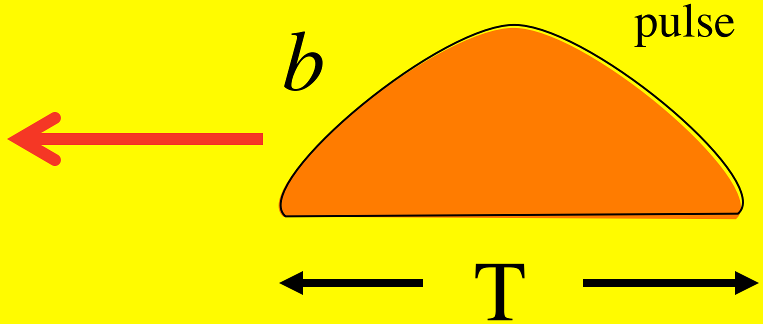
$$V = \sqrt{\omega_p \omega} / 2$$

Malkin, Shvets, and Fisch, (PRL, 1999)

$$a \equiv \frac{eA_{\text{pump}}}{m_e c^2}, \quad b \equiv \frac{eA_{\text{pulse}}}{m_e c^2},$$

f is normalized plasma wave amplitude

$$\omega >> \omega_p$$



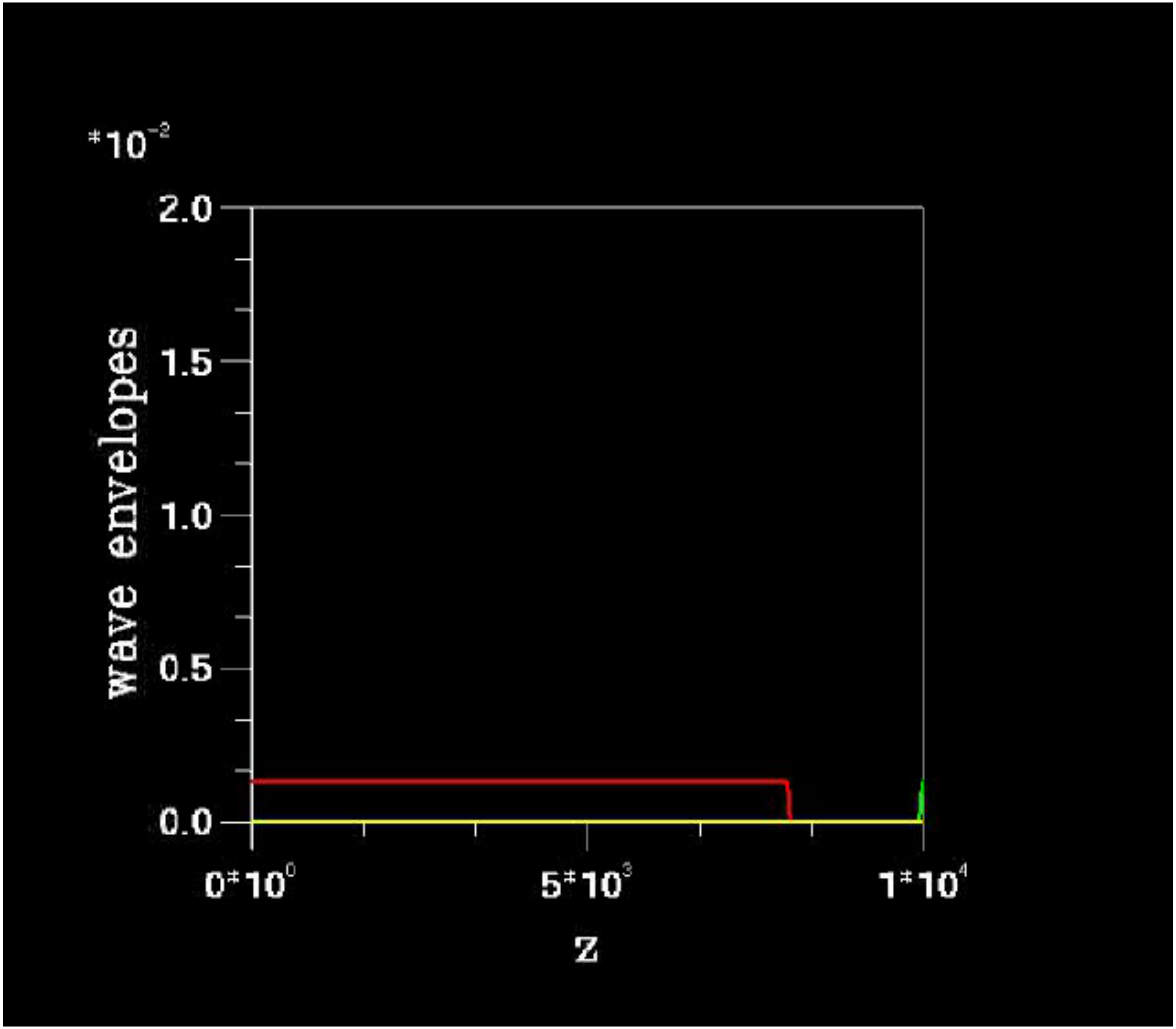
Self-similar solutions:

$b \sim t$

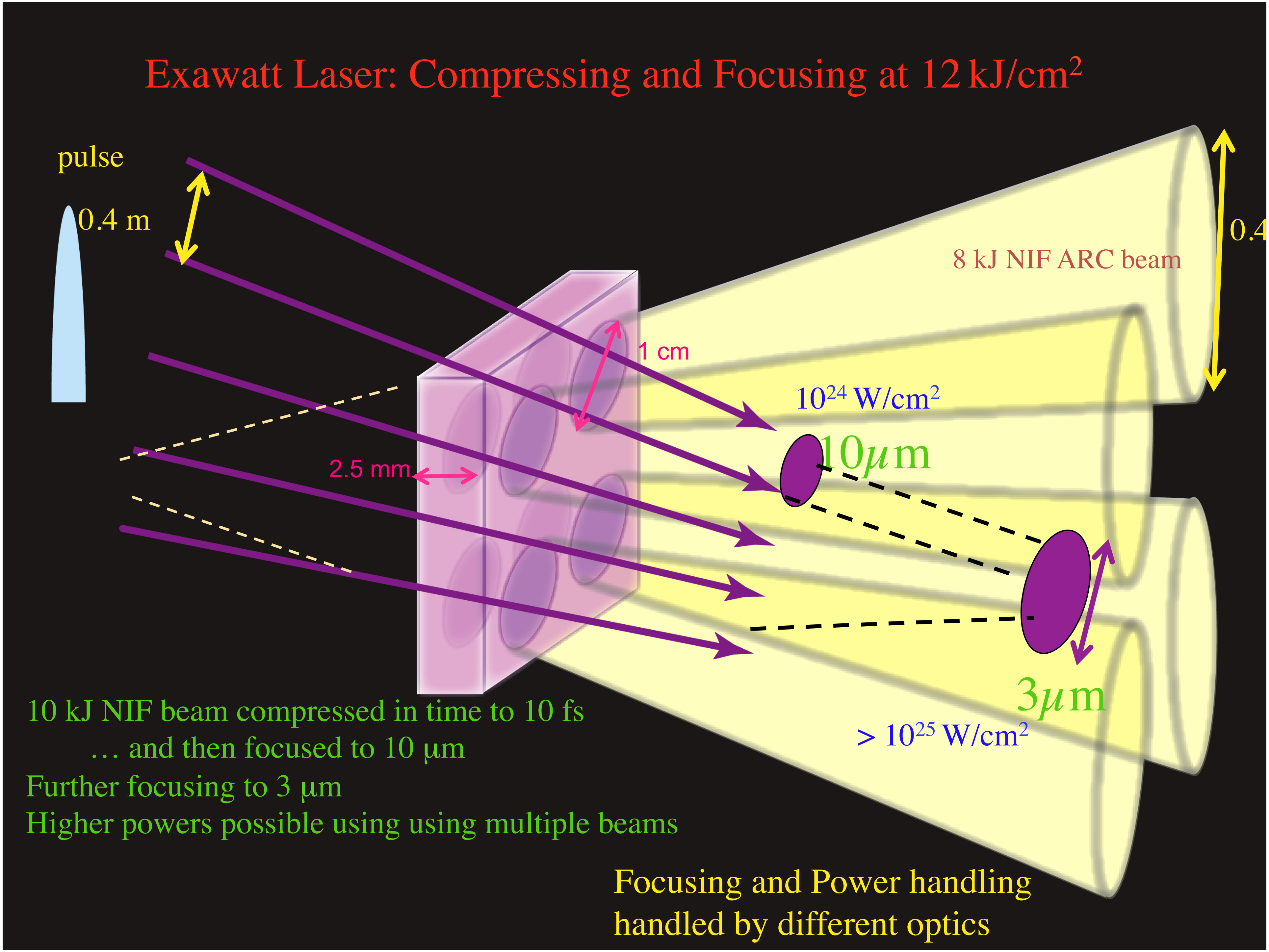
$T \sim 1/t$

$E = b^2 T \sim t$

$z \sim t$

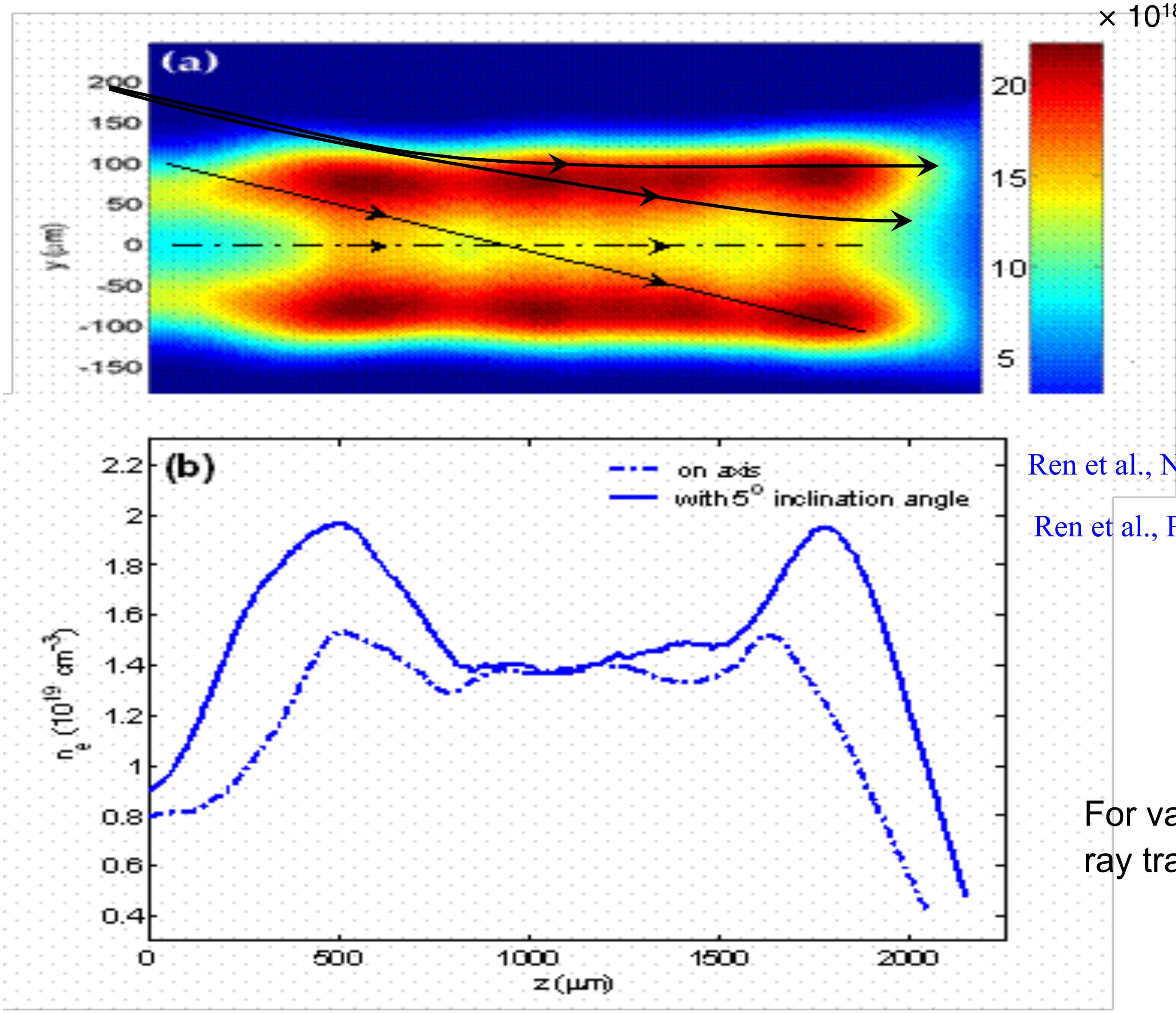


*10^-2
2.0
1.5
1.0
0.5
0.0
wave envelopes
0*10^0
5*10^3
1*10^4
z


Exawatt Laser: Compressing and Focusing at 12 kJ/cm²
pulse
0.4 m
0.4
8 kJ NIF ARC beam
1 cm
2.5 mm
10²⁴ W/cm²
10μm
3μm
> 10²⁵ W/cm²
10 kJ NIF beam compressed in time to 10 fs
… and then focused to 10 μm
Further focusing to 3 μm
Higher powers possible using using multiple beams
Focusing and Power handling
handled by different optics

Tilted Laser Experiments

Suckewer (2007)

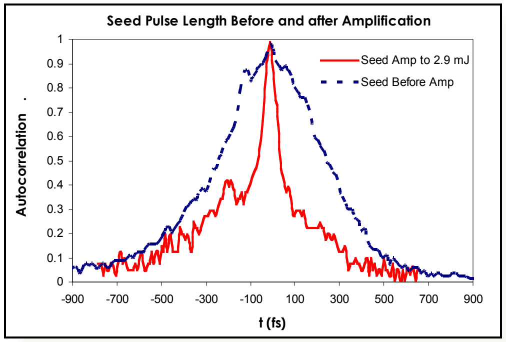


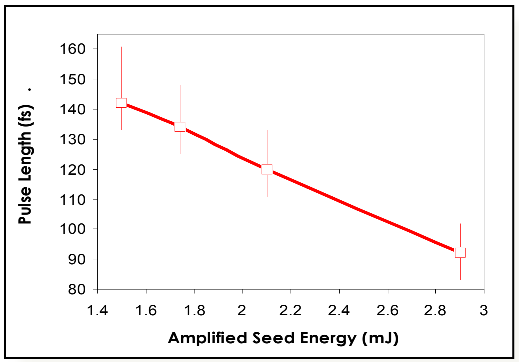
Ren et al., Nature Physics (2007)

Ren et al., POP (2008)

For vacuum
ray trajectory

Signatures of nonlinear self-similar regime





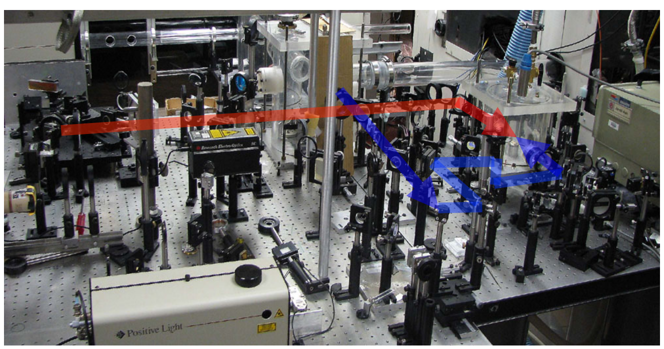
Factors of 100 in seed intensity over pump intensity

Decreased duration of the amplified pulse (50-90 fs)

Pedestal in the autocorrelation function evidences either precursors or secondary spikes

Duration of the amplified pulse decreases inversely with the pulse energy, as in nonlinear π-pulse solution regime

Suckewer Lab



Pump Seed

Large-- but still lower than theoretical maximum efficiency (up to 6.5%, but more adjusting for temporal and spatial overlap)

Amplification tends to saturate at high pump intensity.

Examples

Wavelength of laser μm	1/40	1/4	1	10
Duration of pump ps	1.25	12.5	50	500
Intensity of pump W/cm^2	1.6×10^{17}	1.6×10^{15}	10^{14}	10^{12}
Pump vector-potential a_0	0.006	0.006	0.006	0.006
Laser-to-plasma frequency ratio	12	12	12	12
Concentration of plasma cm^{-3}	1.1×10^{22}	1.1×10^{20}	7×10^{18}	7×10^{16}
Linear e-times growth length cm	.00043	.0043	.013	.13
Total length of amplification cm	.018	.18	.7	7
Output pulse duration fs	1	10	40	400
Output pulse fluence kJ/cm^2	160	16	4	0.4
Output pulse intensity W/cm^2	1.6×10^{20}	1.6×10^{18}	10^{17}	10^{15}

Laser energy vacuum breakdown

Critical electric field $E_c = 1.3 \times 10^{16}$ V/cm^2

Laser compression and focusing to laser wavelength λ

Energy located within volume λ^3

$$W_c \sim \lambda^3 E_c^{\,2}/8\pi \sim \lambda^3\, 8 \times 10^{18}\ \mathrm{J/cm^3}$$

λ	1 μm	100 nm	10 nm	1nm	1 Å
W_c	8 MJ	8 kJ	8 J	8 mJ	8 μJ

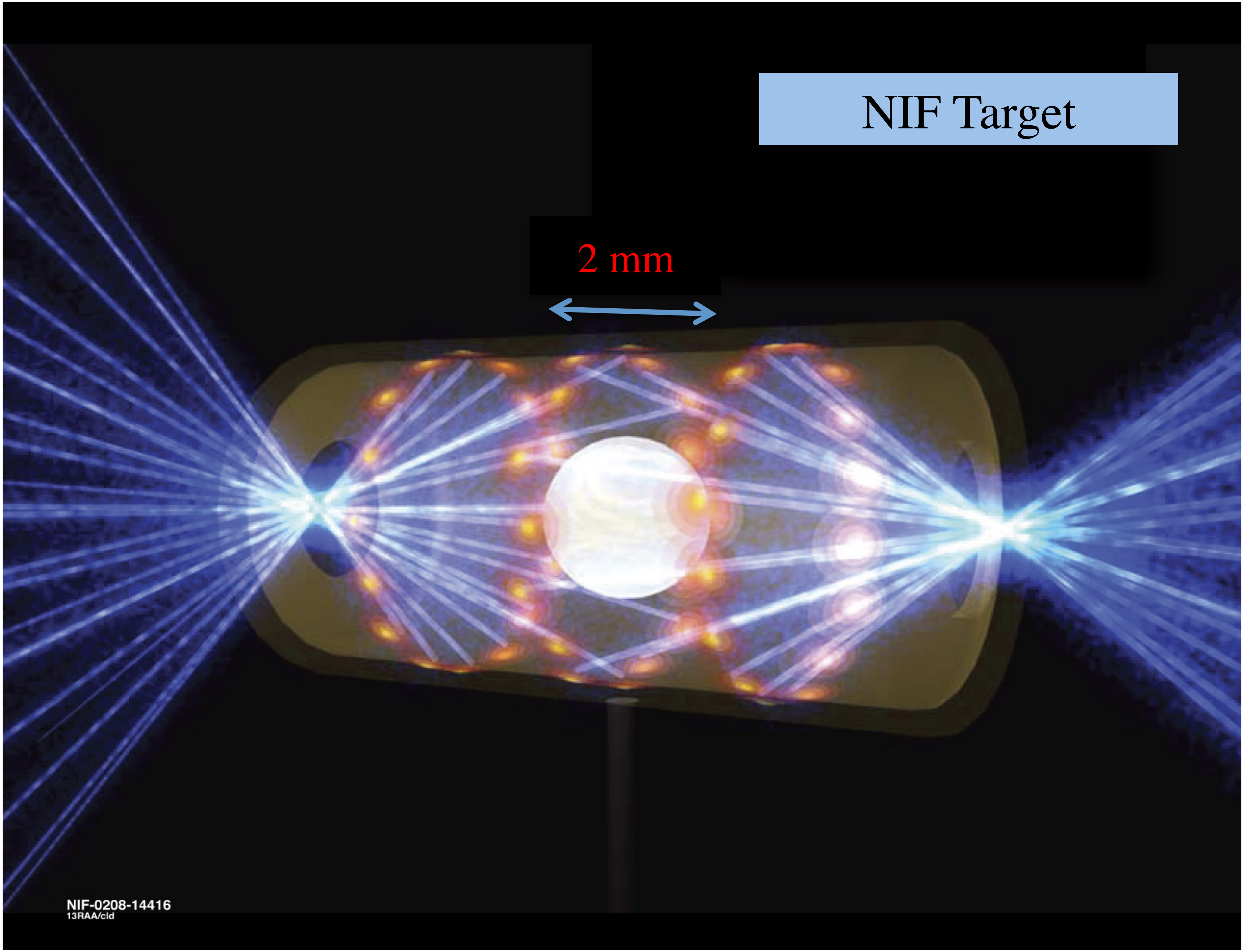
Use MJ optical or mJ x-ray lasers?

Malkin, Fisch and Wurtele (PRE, 2007)

Largest Plasma Compression Facilities

NIF -- National Ignition Facility (LLNL)

Z-Machine (Sandia National Laboratory)


NIF Target
2 mm
NIF-0208-14416
13RAA/cld

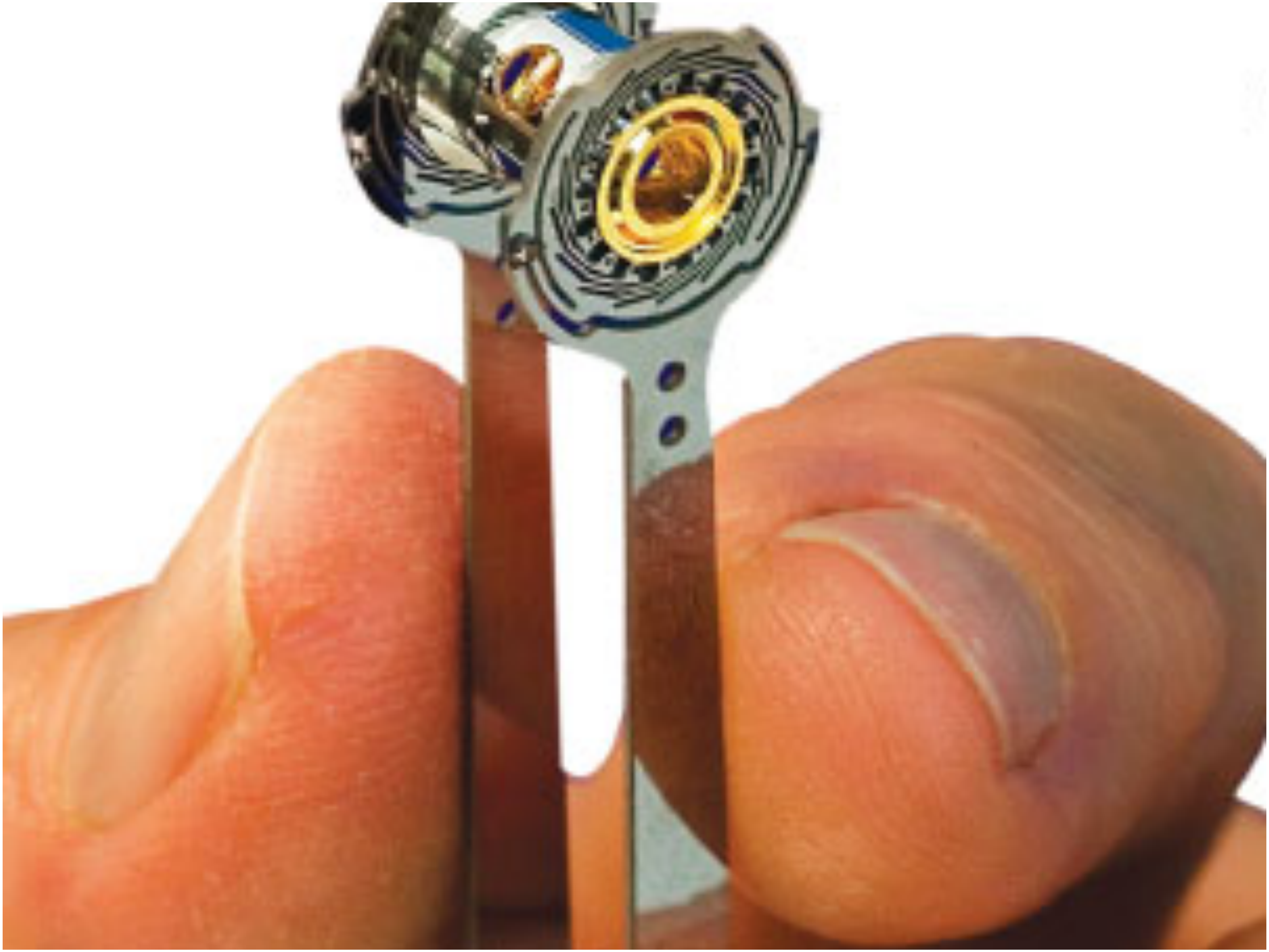
Wave Compression Motivated by New Extreme Compression Facilities

New $B facilities are being built to compress plasma to 1000 times solid density in as little as a nanosecond.

Waves with small group velocity, such as Langmuir waves, can be compressed if the compression time is short compared to the collision time, but long compared to the wave period.

As the imbedded wave grows, the ratio of the field energy to the plasma kinetic energy changes, which can in turn govern a variety of plasma processes.

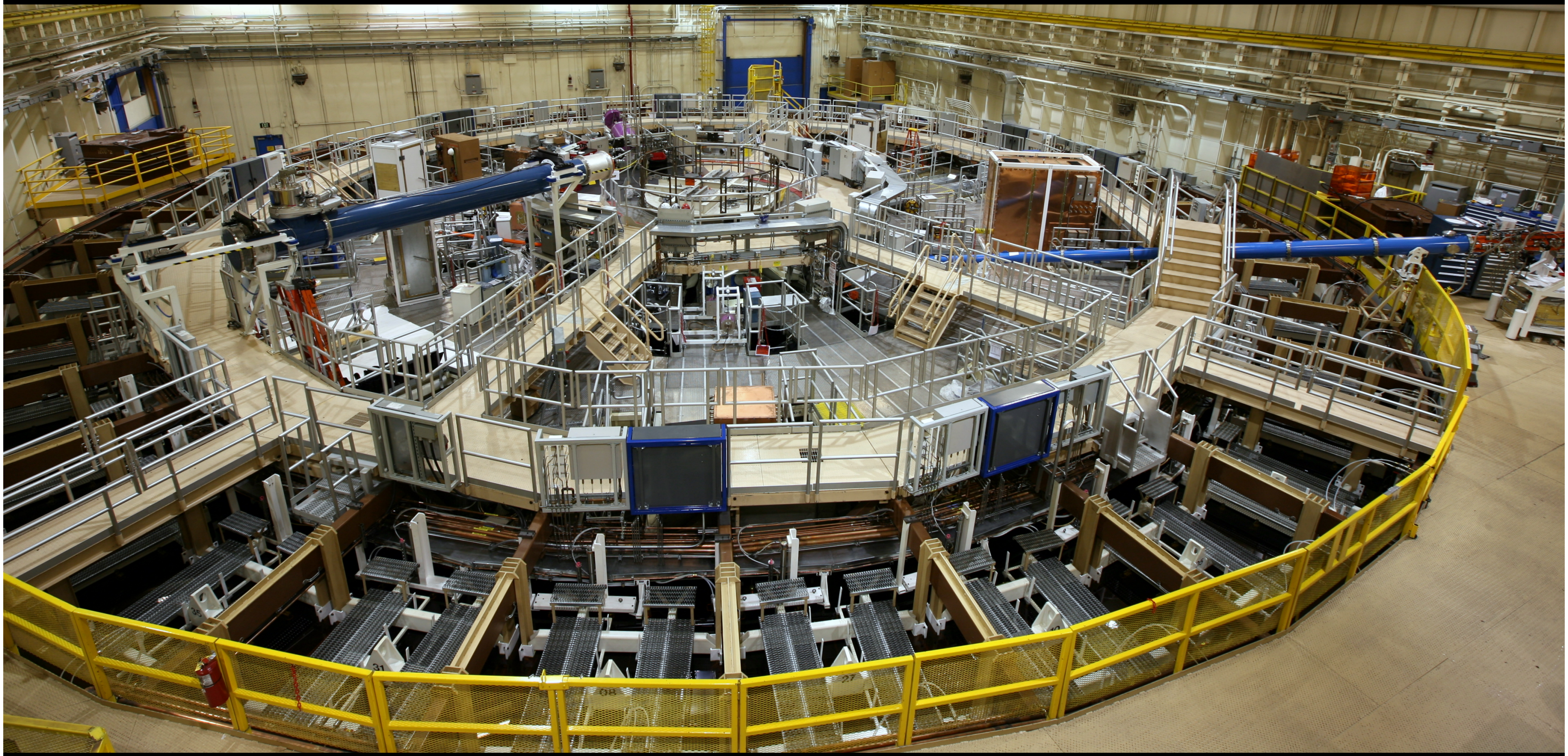
The separate control of wave energy, decoupled from the random particle energy, can be very useful.

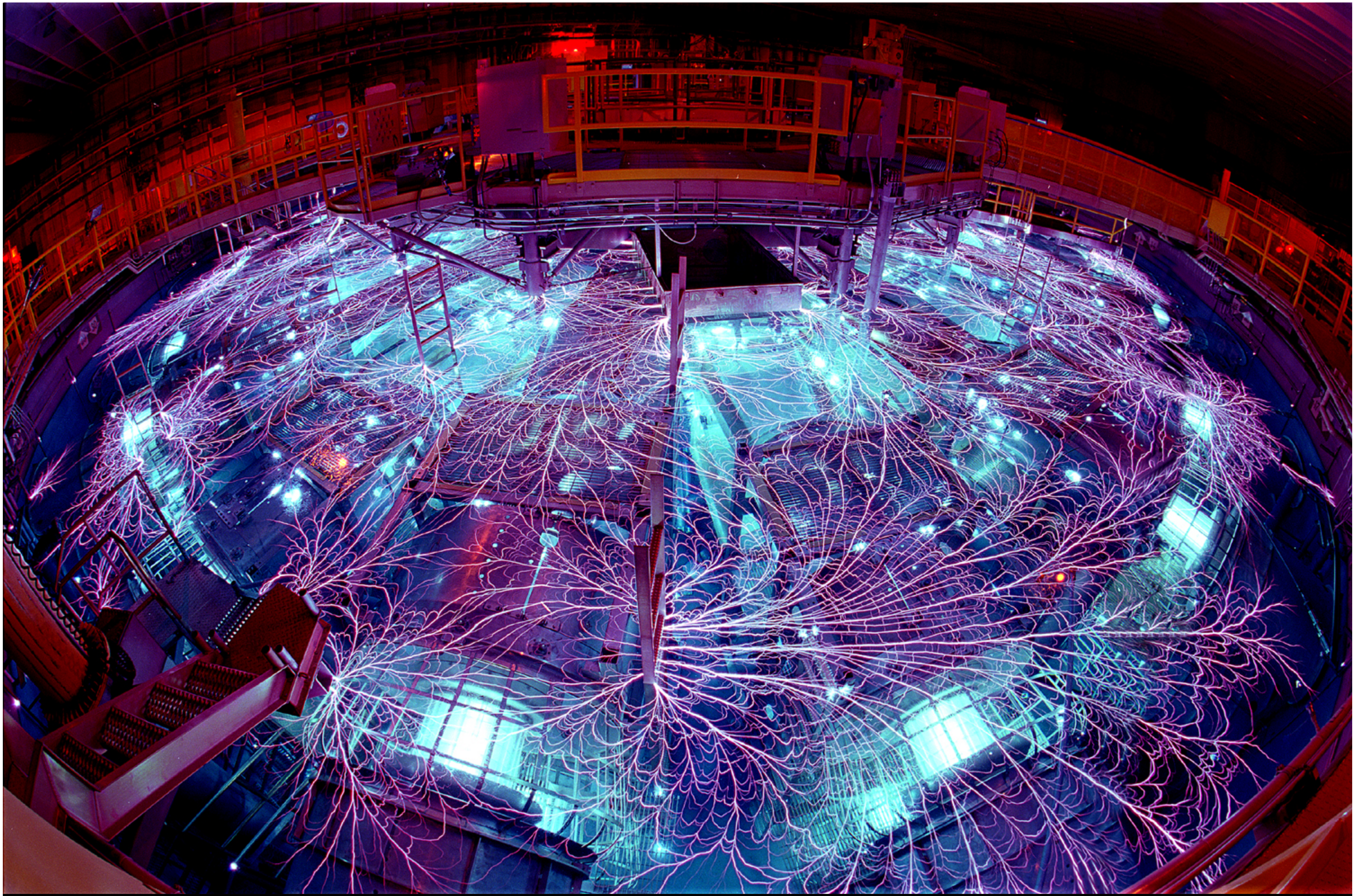



NIF
Target Chamber
NIF-0105-10124
31EIM/dj
P8136


lasers.llnl.gov

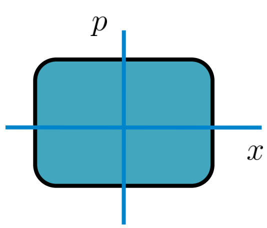
Z-Machine (Sandia)





Adiabatic Compression of Waves (action conservation)

$$\Delta p = 2mU = -2m\frac{\Delta L}{\Delta t} = -\frac{\Delta L}{L}p$$



$$\boxed{\oint p\, dx = \text{inv}}$$

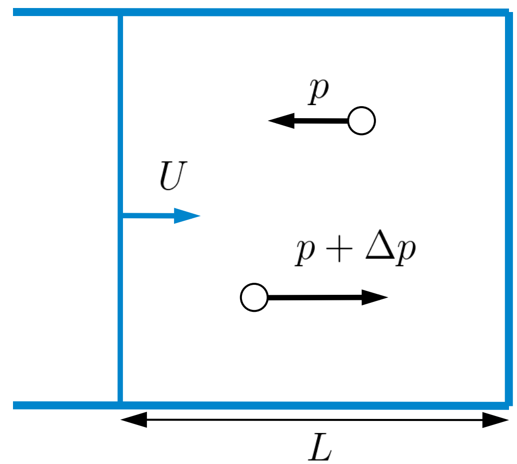
$$\mathcal{E} = Np^2/(2m) \propto L^{-2} \propto V^{-2}$$

- Wave as a number of quanta:

$$\mathcal{E}/\omega = \hbar N = I$$

$$J = I/\mathcal{V}$$

$$\boxed{\partial_t J + \nabla\cdot(\mathbf{v}_g J) = 0}$$



$$E = \hbar\omega \qquad p = \hbar k$$

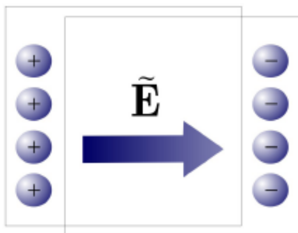
$$\oint p\, dx = \text{inv} \quad \Rightarrow \quad kL = \text{inv}$$

$$\omega = kc$$

$$\mathcal{E} = N\hbar\omega = N\hbar ck \propto L^{-1} \propto V^{-1}$$

Langmuir Wave Compression: Fluid Approach

$$\omega_p^2 = 4\pi N e^2/m_e$$



- Models vary in EOS, or the expression for $\hat{\mathbf{P}}_e$

$$\partial_t N_e + \nabla \cdot (N_e \mathbf{V}_e) = 0$$

$$\partial_t \mathbf{V}_e + (\mathbf{V}_e \cdot \nabla)\mathbf{V}_e = -(e/m_e)\,\nabla\varphi - \nabla \cdot \hat{\mathbf{P}}_e/(N_e m_e)$$

- Don't *assume* EOS; instead, *derive* it from

$$\partial_t \hat{\mathbf{P}}_e + (\mathbf{V}_e \cdot \nabla)\hat{\mathbf{P}}_e + \hat{\mathbf{P}}_e(\nabla \cdot \mathbf{V}_e) + [(\hat{\mathbf{P}}_e \nabla)\mathbf{V}_e] + [(\hat{\mathbf{P}}_e \nabla)\mathbf{V}_e]^{\mathsf{T}} = 0$$

$$\boxed{\begin{aligned}&\frac{\partial'^2 n}{\partial t^2} + \omega_p^2 n - C_{j\ell}\,\frac{\partial^2 n}{\partial x_j\,\partial x_\ell} + \\ &\qquad + 2\,\frac{\partial' n}{\partial t}\left(\frac{\Omega}{\omega_p}\,\frac{\partial \omega_p}{\partial x_\ell} + k_j W_{j\ell}\right)\frac{k_\ell}{k^2} - \left(\delta_{js} + \frac{k_j k_s}{k^2}\right)\frac{\partial C_{s\ell}}{\partial x_j}\,\frac{\partial n}{\partial x_\ell} = 0\end{aligned}}$$

$$\omega^2 = \omega_p^2 + 3k^2 v_{Te}^2 \approx \omega_p^2 \qquad\qquad \mathcal{E} = |E|^2/(8\pi) \propto N^{3/2}$$

Dodin, Geyko, & Fisch, POP, 2009

What happens to imbedded waves as plasma is compressed?

Regime of adiabatic compression: $\boxed{1/\nu < \tau_{comp} < 1/\omega}$

Action conservation: $\dfrac{VE^2}{\omega} \sim const$

Example:
Plasma Waves

$\omega \sim n^{1/2} \sim V^{-1/2}$ $\longrightarrow$ $\boxed{E \sim V^{-3/4} \sim n^{3/4}}$

$P_{pw} = \dfrac{E^2}{16\pi}$ $\longrightarrow$ $\boxed{P_{pw} V^{3/2} = const}$

compare: $PV^{\gamma} = const$

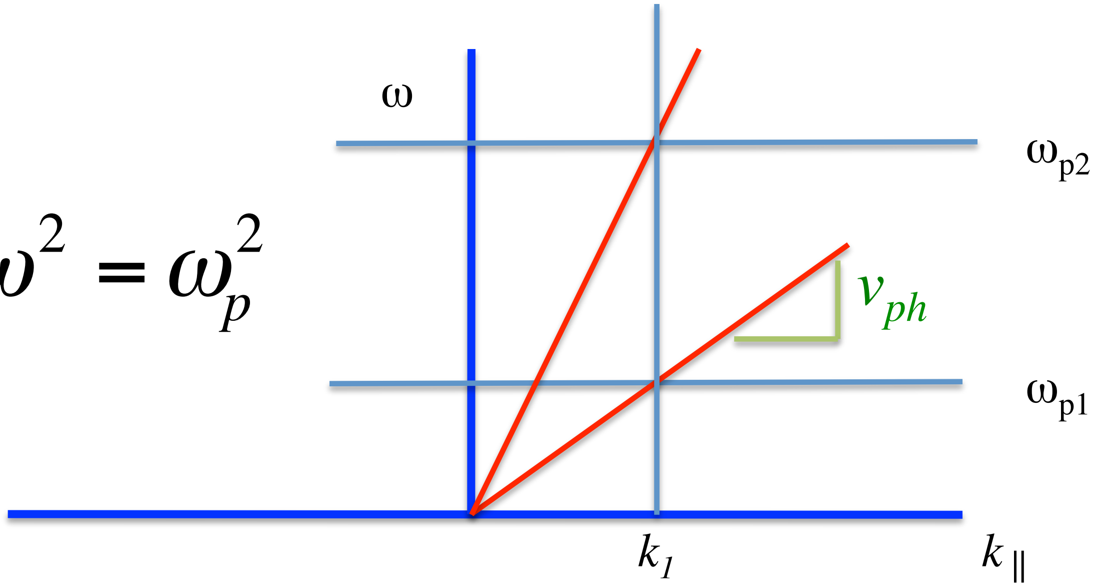
3D: $\gamma = 5/3$

1D: $\gamma = 3$

$\gamma = \dfrac{m+2}{m}$

Compression Perpendicular to *k*

$$\omega^2 = \omega_p^2$$



$\omega_{p1} \longrightarrow \omega_{p2}$

$k_1 \longrightarrow k_1$

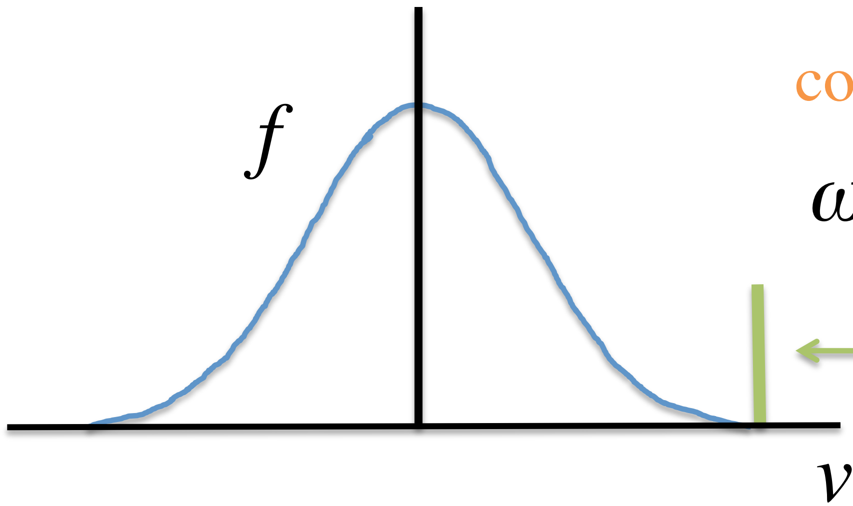
Under compression: Less damping,

if collisionless $\longrightarrow$ $T_\perp > T_\parallel$

Under expansion: More damping, $T_\perp < T_\parallel$

Extra wave energy $\longrightarrow$ $T_\parallel$

Current Drive and Heating

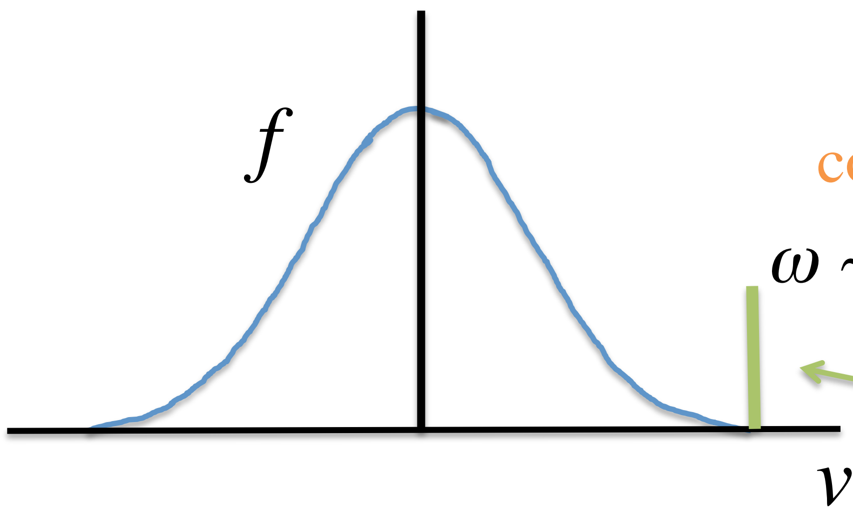


compression perpendicular to k

$$\omega \sim n^{1/2} \qquad k \sim v_{\parallel} \sim const$$

$$\boxed{\omega / k v_{\parallel} \sim n^{1/2}}$$

Note: under expansion, $T_{\perp} < T_{\parallel}$



compression parallel to k

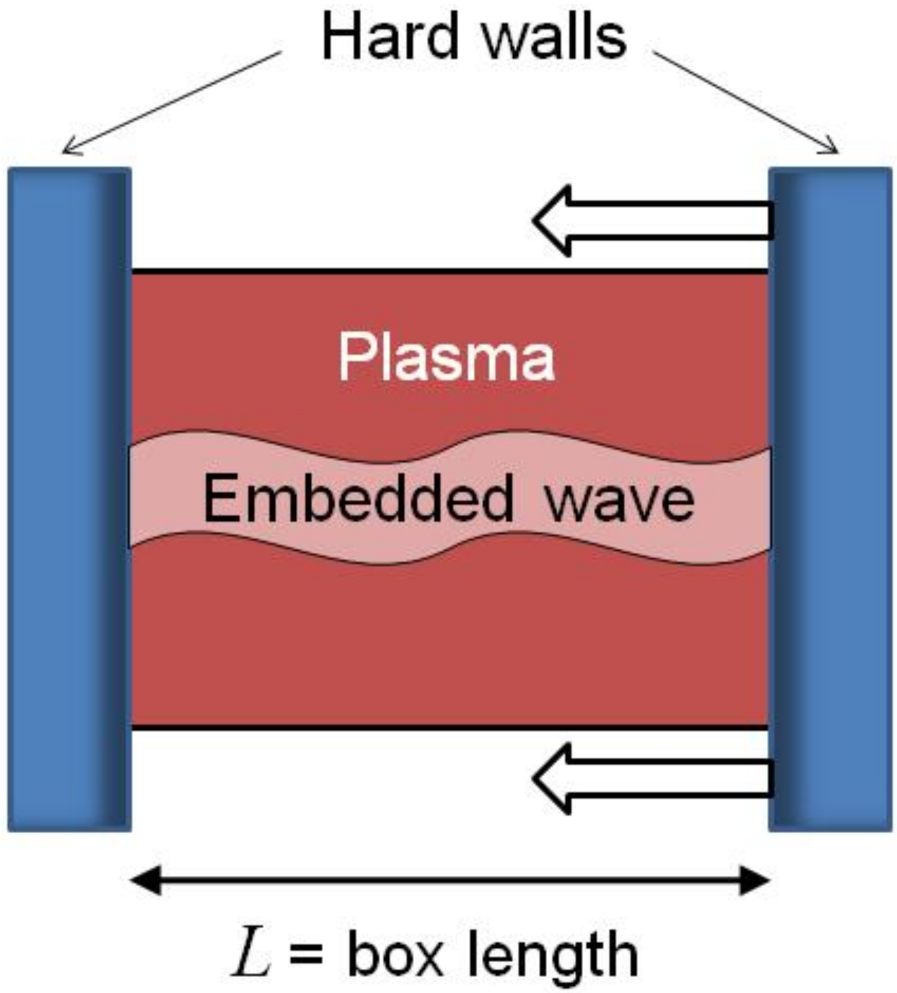
$$\omega \sim n^{1/2} \qquad k \sim 1/L \sim n \qquad v_{\parallel} \sim 1/L \sim n$$

$$\boxed{\omega / k v_{\parallel} \sim n^{-3/2}}$$

Note: under compression, $T_{\perp} < T_{\parallel}$

In either case, extra wave energy can accentuate energy difference

Particle Simulations



PIC simulation schematic

Schmit, Dodin and Fisch, PRL 2010

Langmuir Wave "Switch"

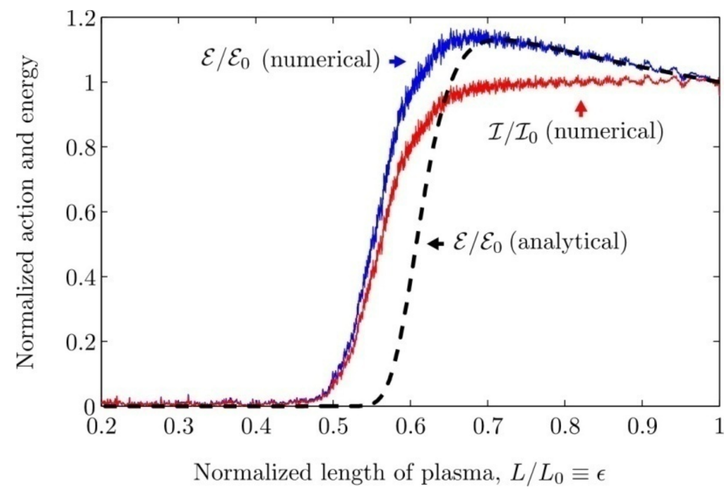
$$\boxed{\omega^2 = \omega_p^2 + 3k^2 v_{Te}^2 \approx \omega_p^2(n)}$$

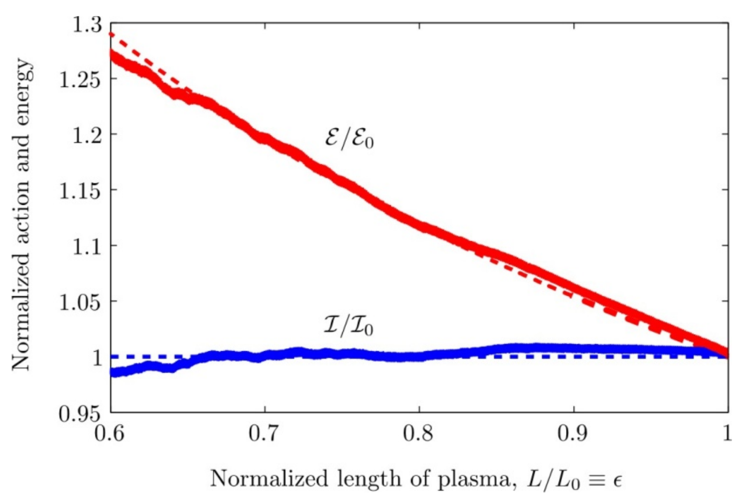
$$|E| \propto n^{3/4}$$

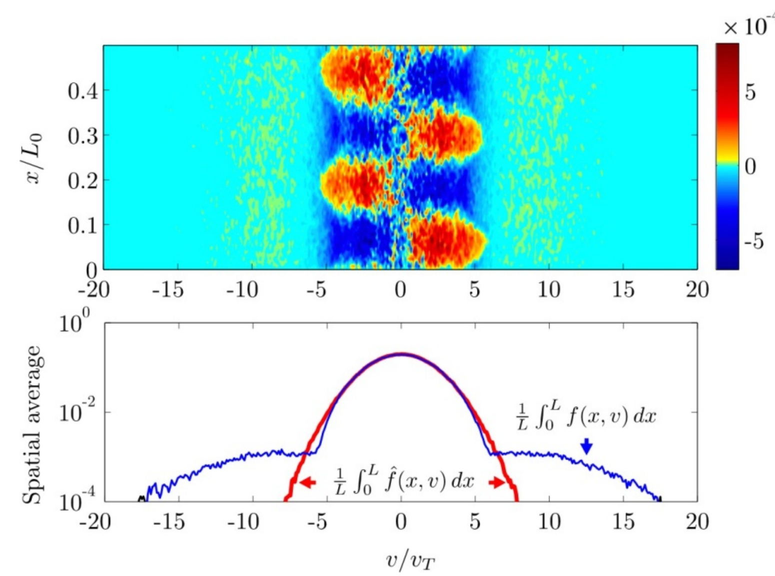
$$k\lambda_{\mathrm{D}} \propto L^{-3/2}$$

Dodin, Geyko, and Fisch, Phys. Plasmas (2010)

Schmit, Dodin, and Fisch, PRL (2010)







What can compressed waves usefully do?

1. Drive current

2. Reduce heat conduction

3. Heat ions preferentially

4. Slow down alpha particles

5. Reduce plasma pressure (increase compressibility)

6. All of the above, but with precise control in time

Summary

Some uses of a Plasma Wave

1. High-gradient accelerators
2. Toroidal current in tokamaks
3. Mediate resonant Raman compression of optical lasers in plasma
4. Compression of x-rays, short wavelength optical
5. Switch-like mechanisms in compression of plasma waves

Goal: Achieve next generation of light intensities

Goal: Realize new (timely) effects in new facilities for highly compressing plasma

Goal: Facilitate economical fusion power